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Reducing the Peak to Average Power Ratio of OFDM Signals by Using Harr Wavelet

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Abstract: With the demand for data rate and dependability in devices and wireless communications growing, various issues—such as bandwidth efficiency, quality of service, and radio coverage—become crucial in Orthogonal Frequency Division Multiplexing (OFDM)-based systems. Due to the high peak-to-average power ratio (PAPR) owned by the transmitted signals, OFDM is particularly susceptible to nonlinear influences and does not exhibit resistance to spectral null channels. The Harr wavelet is a mathematical term for a series of downscaled "square-shaped" functions that collectively make up a wavelet family or basis. Similar to Fourier analysis, wavelet analysis enables the representation of a target function over an interval in terms of an orthogonal function basis. The data symbol sequence is decomposed via the Haar wavelet processing.

By analysing, enhancing, and implementing the Cumulative Distribution Functions of both the Conventional and Proposed OFDM, it can be seen that the Proposed OFDM performs better than the Conventional OFDM.

Keywords- Peak-to-average power ratio (PAPR); Haar Wavelet; Orthogonal frequency division multiplexing (OFDM).

1. INTRODUCTION

One of the numerous multicarrier modulation systems is OFDM, which offers excellent spectral efficiency, low implementation complexity, decreased sensitivity to echoes and non-linear distortion, and is also relatively simple to implement. The OFDM system is widely employed in numerous communication systems as a result of these benefits. However, the high peak-to-average power ratio of this system is the main issue that arises while applying it. A big PAPR decreases the radio frequency (RF) power amplifier's efficiency and adds to the complexity of the analog-to-digital and digital-to-analog converters. It is possible to implement regulatory and application limits to lower the peak transmitted power, which in turn lowers the multi carrier transmission's range.

The transmitter power amplifier is no longer restricted to the linear region in which it should operate, which prevents spectrum expansion. The battery life suffers as a result of this. Thus, in communication systems, it is seen that a high PAPR value has the potential to outweigh all of the advantages of multi carrier transmission. An OFDM system's peak value can be significantly higher than the system average if there are a lot of individually modulated sub-carriers present.

Peak-to-Average Power Ratio is the name given to this relationship between peak and average power values. A peak that is N times the average signal is produced by the coherent addition of N signals with the same phase. Without lowering peaks, it is difficult to transmit signals with such high peak amplitudes to the transmitter. Therefore, before broadcasting, we must lower the signals' high peak amplitude.

The major disadvantages of a high PAPR are-

1. Increased complexity in the analog to digital and digital to analog converter.
2. Reduction is efficiency of RF amplifiers.

The peak to average power ratio for a signal x(t) is defined as

$$\text{Papr} = \frac{max\left|x(t)^2\right|}{E\{|x(t)^2|\}}$$

Where $()^*$ corresponds to conjugate operator

Expressing in decibels

$$\text{Papr (db)} = 10 \log_{10} (\text{Papr})$$

2. PAPR REDUCTION TECHNIQUES

The methods used to reduce PAPR differ depending on the system's requirements and a number of other variables. Before implementing a PAPR reduction technique for the system, numerous aspects like PAPR reduction capacity, increase in transmit signal power, loss in data rate, complexity of calculation, and rise in bit-error rate at the receiver end are taken into consideration. The following list of PAPR reduction methods includes:

- **Amplitude Clipping and Filtering: -** In this process, an amplitude threshold value is specified, and any sub-carrier with amplitude greater than that value is clipped or filtered to produce a reduced PAPR result.

- **Selected Mapping**: - In this, a group of data blocks that are sufficiently diverse from one another and that represent the same information as the original data blocks are chosen. It is suitable for transmission when data blocks with low PAPR values are chosen.

- **Partial Transmit Sequence: -** It is the process of sending only a portion of the data, which includes all the information that needs to be delivered in the signal as a whole but is carried by a different sub-carrier.

3. PROPOSED OFDM TRANSMITTER AND RECEIVER:

Below figure provides a descriptive description of a fundamental OFDM modulator. Each modulator in the bank of N complex modulators corresponds to a different OFDM subcarrier. Maps outline the modulation strategy (Binary phase-shift keying or BPSK, QPSK, 16-QAM ...). The information obtained is the frequency-dependent amplitude and phase of each sub-carrier. We are able to determine the in-phase and quadrature components of the related time-domain waveform by subjecting it to an inverse fast fourier transform (IFFT).

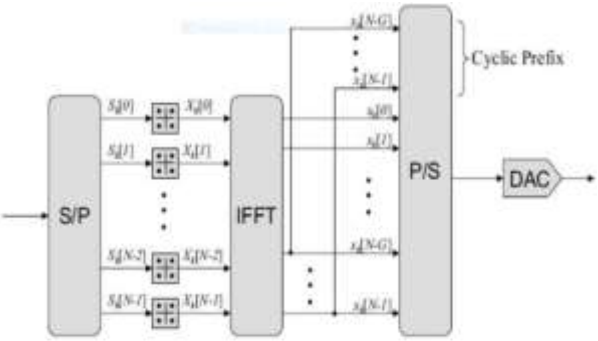

Fig 3.1 OFDM transmitter block diagram

LTE employs a little more intricate method called Cyclic Prefix (CP) insertion. As before, the LTE transmitter begins by adding a guard period before each symbol. To fill the guard period, it then copies information from the end of the symbol that comes after. In actuality, the CP is a copy of the conclusion of a symbol that was added at the start. Inter-symbol interference can be minimised if the guard period is longer than the delay spread in the radio channel and each OFDM symbol is cyclically extended into the guard period (by duplicating the symbol's end to its beginning to generate the cyclic prefix). The information can ultimately be combined to radio frequency for transmission.

In the receiver the reverse operations are done to extract the originally sent message. But there should be some extra blocks to make more reception such as synchronization and equalization. Synchronization is done to overcome frequency and time set.

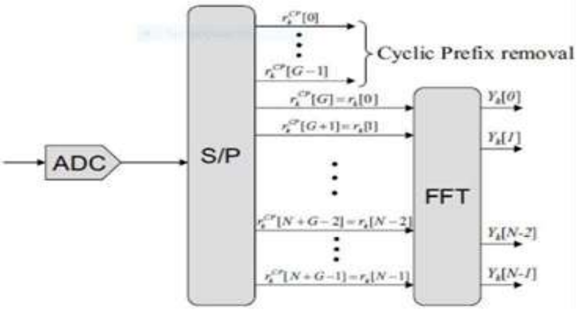

Fig 3.2 OFDM receiver block diagram

4. OFDM GENERATION AND RECEPTION:

The relationship between each carrier must be carefully managed to maintain the carriers' orthogonality in order to generate OFDM properly. Due to this, the generation of OFDM begins with the selection of the necessary spectrum depending on the input data and modulation technique. Some data is designated to transmit on each carrier that will be created. The modulation strategy is then used to compute the necessary carrier amplitude and phase (typically BPSK, QPSK, or QAM). An Inverse Fourier Transform is then used to return the needed spectrum to its time domain signal. An Inverse Fast Fourier Transform (IFFT) is employed in the majority of applications. The IFFT performs the transformation effectively and offers a straightforward method of guaranteeing that the carrier signals generated are orthogonal.

A cyclic time domain signal is transformed into its equivalent frequency spectrum using the Fast Fourier Transform (FFT). Finding the equivalent waveform produced by adding orthogonal sinusoidal components is how this is accomplished. The frequency spectrum of the time domain signal is represented by the amplitude and phase of the sinusoidal components. The reverse process is carried out by the IFFT, which converts a spectrum (the amplitude and phase of each component) into a time domain signal. A number of complicated data points with lengths that are powers of two are converted by an IFFT into a time domain signal with the same number of points. The term "bin" refers to each frequency spectrum data point utilised in an FFT or IFFT.

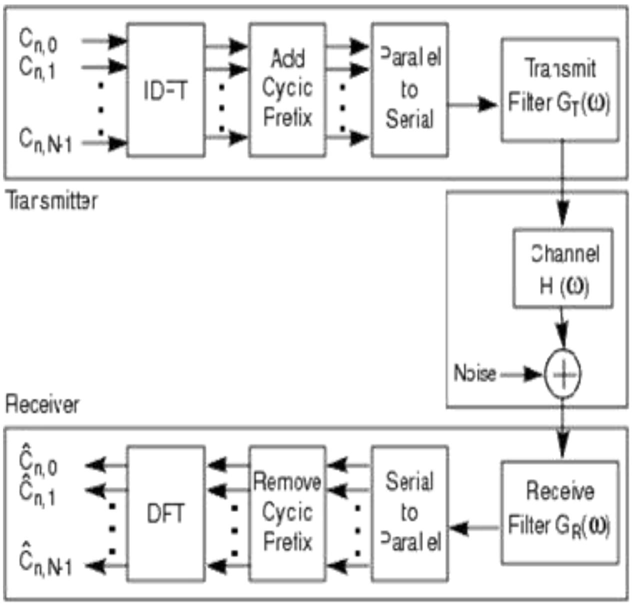

Fig 4.1 OFDM Block diagram

5. PEAK TO AVERAGE POWER RATIO IN OFDM SYSTEM:

If we consider N modulated data symbols from a particular signalling constellation, $X_k = (X_0, X_1, \ldots\ldots X_{N-1})$, over a time interval $[0, T]$, the OFDM symbol can be written as

$$x(t) = \sum_{k=0}^{N-1} x_k e^{j2\prod kf0t}$$

where: $f_0 = 1/T$. Replacing $t = nTb$, where $Tb = T/N$, we arrive at the discrete time version given by:

$$x_n = \sum_{k=0}^{N-1} x_k e^{j2\prod kn/N}$$

The PAPR of the signal, $x(t)$, is then given as the ratio of the peak instantaneous power to the average power, written as:

$$\text{PAPR} = \frac{\max\,[x(t)\ x*(t)]}{E[x(t)x*(t)]}$$

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$$\text{PAPR}=\frac{\max\,[x(t)\,x*(t)]}{E[x(t)x*(t)]}$$

7. HAAR WAVELET TRANSFORM:

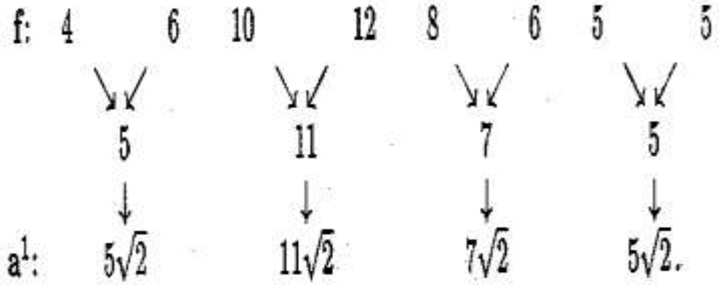
Alfred Haar, a Hungarian mathematician, proposed the Haar transform, one of the first transform functions, in 1910. It is effective because it offers a straightforward method for analysing the local characteristics of a signal. The first and most basic wavelet is called a Haar wavelet. The continuous Haar wavelet resembles a step function. The wavelet it represents is Daubechies db1's. We must first specify the kinds of signals that the Haar transform will be used to analyse. We will use discrete signals extensively throughout this book. A discrete signal is one that has values that change throughout time at specific moments. A discrete signal is typically expressed in the form f = (f1, f2,..., fN), where N is an even positive integer we'll refer to as the length of f. The N real numbers f1, f2,..., fN are the values of f. Typically, these values are measurements of an analogue signal g taken at time values t = t1, t2,..., tN. That is, f1 = g(t1), f2 = g(t2),..., fN = g (tN) are the values of f. For the sake of simplicity, we'll suppose that each pair of subsequent time values is always separated by the same amount of time. When the discrete signal's values are defined, we'll say "equally spaced sample values" or "only sample values."

The set of data values included in a computer audio file, such as a wav file, is a crucial illustration of sample values. The sound intensity readings on a CD are another illustration. A digitised ECG is an example of a non-audio signal, where the analogue signal g is not a sound signal.

The Haar transform divides a discrete signal into two smaller signals that are each half its length, just like all other wavelet transforms do. A running average, trend, or fluctuation is one sub signal, while a running difference, fluctuation, is the other. Start by looking at the trend sub signal. The first trend sub signal for the signal f is calculated by taking a running average in the manner shown below: a1 = (a1, a2,..., aN/2).Its first value, *a*1, is computed by taking the average of the first pair of values of **f**: (*f*1 + *f*2)/2, and then multiplying it by$\sqrt{2}$. That is, *a*1 = (*f*1+*f*2)/$\sqrt{2}$. Similarly, its next value *a*2 is computed by taking the average of the next pair of values of **f**: (*f*3 +*f*4)/2, and then multiplying it by$\sqrt{2}$. That is, *a*2 = (*f*3 + *f*4)/$\sqrt{2}$. Continuing in this way, all of the values of **a**1 are produced by taking averages of successive pairs of values of **f** and then multiplying these averages by$\sqrt{2}$. A precise formula for the values of **a**1 is

$$a_m = \frac{f_{2m-1}+f_{2m}}{\sqrt{2}}, m=1,2,3,\ldots N/2$$

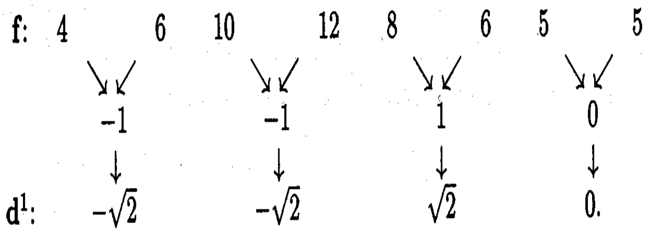
For example, suppose **f** is defined by eight values, say **f** = (4, 6, 10, 12, 8, 6, 5, 5); then its first trend sub signal is **a**1 = (5$\sqrt{2}$, 11$\sqrt{2}$, 7$\sqrt{2}$, 5$\sqrt{2}$). This result can be obtained using Formula (2.7.1). Or it can be calculated as indicated



The other sub signal is called the *first fluctuation.* The first fluctuation of the signal **f**, which is denoted by **d** = (d_1, d_2, . . .,$d_{N/2}$), is computed by taking a running difference in the following way. Its first value, $d1$, is calculated by taking half the difference of the first pair of values of **f** :($f1-f2$)/2, and multiplying it by $\sqrt{2}$. That is, $d_1 = (f1-f2)/\sqrt{2}$. Likewise, its next value $d2$ is calculated by taking half the difference of the next pair of values of **f**: ($f3 - f4$)/2, and multiplying it by $\sqrt{2}$. In other words, $d_2 = (f3 - f4)/\sqrt{2}$. Continuing in this way, all of the values of **d**1 are produced according to the following formula.

$$d_m = \frac{f_{2m-1}-f_{2m}}{\sqrt{2}}, \qquad m=1,2,3,N/2$$

For example, for the signal **f** = (4, 6, 10, 12, 8, 6, 5, 5) considered above, its first fluctuation **d**1 is ($-\sqrt{2}$,$-\sqrt{2}$,$\sqrt{2}$, 0). This result can be obtained using Formula (2.7.2), or it can be calculated as indicated in the following diagram:



The Haar wavelet's mother wavelet function $\psi(t)$can be described as

$$\boldsymbol{\psi(t)} = \begin{cases} \mathbf{1}, & \boldsymbol{for\ 0 \le t < \frac{1}{2}} \\ \mathbf{-1}, & \boldsymbol{for\ \frac{1}{2} \le t < 1} \\ \mathbf{0}, & \boldsymbol{otherwise} \end{cases}$$

Its scaling function $\phi(t)$can be described as

$$\phi(t) = \begin{cases} 1, & for\ 0 \le t < 1 \\ 0, & otherwise \end{cases}$$

8. SIMULATION RESULTS:

The proposed OFDM improves the PAPR statistics of OFDM. When the probability of input symbol ‘1’ or ‘0’ is 0.5, the proposed OFDM approximately has the same cumulative distribution functions as the conventional OFDM. When the probability of input symbol ‘1’ or ‘0’ larger than 0.6, Fig 8.3 shows the proposed OFDM almost has the PAPR reduced by 3 dB approximately

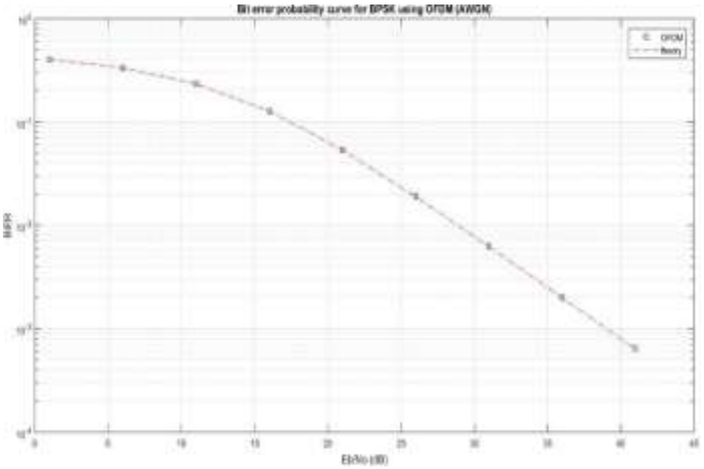


Fig 1: $\boldsymbol{{}^{Eb}/_{No}}$ ***Vs*** Bit error Rate of BPSK over AWGN

The peak power of the Haar wavelet-based BPSK OFDM system is reduced by half, compared with the conventional OFDM system. So, Haar wavelet-based BPSK OFDM system’s peak-average power ratio (PAPR) is reduced by 10 $\log_{10}2\approx$ 3 dB atmost, compared with the conventional OFDM system. Hence, the proposed Haar wavelet-based BPSK OFDM system is able to overcome the drawback of conventional OFDM system, with lower peak-average power ratio.

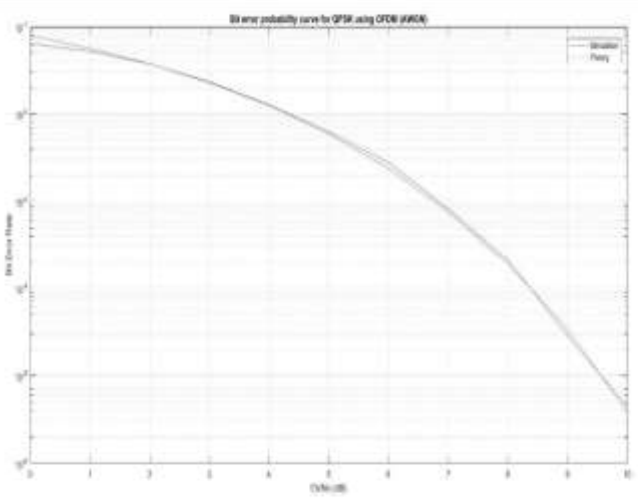


Fig 2: $\boldsymbol{{}^{Eb}/_{No}}$ ***Vs*** Bit error Rate of QPSK over AWGN

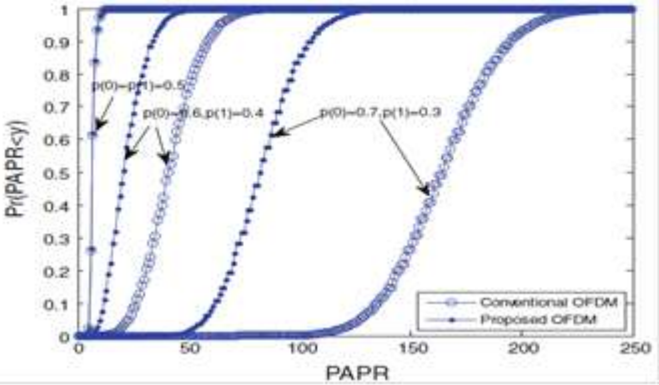


Fig 3: Cumulative Distribution function for Proposed OFDM and Conventional OFDM

The Fig 3 shows the cumulative distribution functions of the PAPR, determined empirically over one million OFDM symbols. Specifically, when the probability of input symbol “1” (after BPSK modulation, it is “−1”) or “0” (after BPSK modulation, it is “1”) is 0.5, a high peak value of OFDM symbols appears with a low probability due to the peak values from different subcarriers can cancel each other. When the probability of “0” or “1” becomes larger or smaller, a lot of symbols with the same phase will add together and produce a larger peak value (thus higher PAPR). Moreover, the PAPR value of OFDM system is also proportional to the number of subcarriers. Our theoretical analysis shows that the PAPR value of our proposed OFDM. A Novel Haar Wavelet-Based BPSK OFDM System can reduce PAPR by $10 \log_{10} 2 \approx 3$dB at most, compared with the conventional OFDM.

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